10-6 Theoretical Probability

Objectives

Determine the theoretical probability of an event.
Convert between probabilities and odds.

The theoretical probability of an event is the ratio of the number of ways the event can occur to the total number of equally likely outcomes.

Theoretical Probability

\[
\text{Theoretical probability} = \frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}
\]

An experiment in which all outcomes are equally likely is said to be fair. You can usually assume that experiments involving coins and number cubes are fair.

An experiment consists of rolling a number cube. Find the theoretical probability of each outcome.

- rolling a 5
  \[
  \frac{1}{6}
  \]
- rolling an odd number
  \[
  \frac{1}{3} \div \frac{3}{6} = \frac{1}{2}
  \]
- rolling a number less than 3
  \[
  \frac{1}{2} - \frac{2}{3} - \frac{1}{3}
  \]

When you toss a coin, there are two possible outcomes, heads or tails. The table below shows the theoretical probabilities and experimental results of tossing a coin 10 times.

<table>
<thead>
<tr>
<th>Event</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heads</td>
<td>[ \frac{1}{2} ]</td>
</tr>
<tr>
<td>Tails</td>
<td>[ \frac{1}{2} ]</td>
</tr>
</tbody>
</table>

The sum of the probability of heads and the probability of tails is 1, or 100%. This is because it is certain that one of the two outcomes will always occur.

\[
P(\text{event happening}) + P(\text{event not happening}) = 1
\]

The complement of an event is all the outcomes in the sample space that are not included in the event. The sum of the probabilities of an event and its complement is 1, or 100%, because the event will either happen or not happen.

\[
P(\text{event}) + P(\text{complement of event}) = 1
\]
1. What are the odds of rolling a 2 on a number cube?

2. The odds in favor of winning a free drink are 3:4. What is the probability of winning a free drink?

3. What are the odds against rolling a 3 or a 4 on a number cube?

4. The odds in favor of Mike wearing a blue shirt are 2:7. What is the probability of him wearing a blue shirt?

- The two numbers given as the odds will add up to the total number of possible outcomes. You can use this relationship to convert between odds and probabilities.

- Odds are another way to express the likelihood of an event. The odds in favor of an event describe the likelihood that the event will occur. The odds against an event describe the likelihood that the event will not occur.

- Odds are usually written with a colon in the form a:b, but can also be written as a to b or \( \frac{a}{b} \).

- **Odds in Favor of an Event**
  
  \[
  \text{odds in favor} = \frac{\text{number of ways an event can happen}}{\text{number of ways an event can fail to happen}} = \frac{a}{b}
  \]

- **Odds Against an Event**
  
  \[
  \text{odds against} = \frac{\text{number of ways an event can fail to happen}}{\text{number of ways an event can happen}} = \frac{b}{a}
  \]

- a represents the number of ways an event can occur.
- b represents the number of ways an event can fail to occur.

- A jar has green, blue, purple, and white marbles. The probability of choosing a green marble is 0.3, the probability of choosing purple is 0.1. What is the probability of choosing white?

- Assignments: p. 723 #1-28, 40-45